1 Miyashita Group

Research Subjects: Statistical Mechanics, Phase Transitions, Quantum Spin systems,

Quantum Dynamics, Non-equilibrium Phenomena

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1.1 Cooperative Phenomena and Phase Transition

Study on phase transitions and critical phenomena is one of main subjects of the statistical mechanics. We have studied various types of ordering phenomena in systems with large fluctuation. In the last year, we studied the following topics of phase transitions.

Phase transitions of long-range interacting systems

Systems with bistable local electric states, such as the spin-crossover, Jahn-Teller system, and martensite systems, have been attracted interests as seminal candidates of the so-called functional material because the bistable states can be switched by the temperature, pressure, magnetic field, and photo-irradiation. We have proposed a general structure of the ordered states including metastable state, where we find various new types of phase transitions.We also pointed out that difference of local structures of the lattice of the states causes a new aspect of the ordering phenomena. In the spin-crossover systems, the size of molecules in the high spin (HS) and low spin (LS) are different and the lattice distorts in the mixture of the both spin states.This lattice distortion causes an effective long range interaction among spin states, and realizes a phase transition of the mean-field universality class.The long range interaction prefers a uniform configuration and thus the systems keeps homogeneous configuration even near the critical temperature. However, when the systems change between the two states in open boundary condition, the systems show inhomogeneous structures.In a rectangle lattice, the changes start from the corners, but the domains which appear in the process are macroscopic. That is, the configurations are the same if we scale the sizes. We also studied the switch in a circular system which has no corner. In this system, a kind of nucleation occurs from the surface. Here we again find that shapes of the critical nuclei and also the following clusters growth are geometrically similar in systems of different sizes. This feature is qualitatively different from that of short-range interaction systems, in which the critical droplet has a specific size independently of the system size.[6]

We have also studied shape and dynamics of the domain wall. In the short-range model, the width of the domain wall is proportional to the square of the system size L. However, in the long-range model, it is found to be proportional to the system size L, and thus again the shapes are geometrically similar in systems of different sizes. In Fig. 1.1.2, we show the configurations of the domain walls of different sizes.[21]

In the long-range model, configurations with large clusters are suppressed.However, if the short-range interaction is included, it cause a short range correlation. Thus the system shows a finite correlation length at the critical point. We studied a scaling relation of the shift of the critical point from the pure short range model as a function of the strength of the long-range interaction. We also studied a scaling relation of the correlation length at the critical point. We first study these properties in an Ising model of mixture of the nearest-neighbor interaction and infinite range interaction in a fixed lattice.[3] Then we found that the scaling relations work in the elastic model, too.[8]

Ordered states of long-range interacting system

We also studied in which condition systems with long range interaction are described by the mean-field theory. It is expected that in the cases where the interaction is non-additive, where the extensivity is not satisfied and the so-called Kac procedure is necessary, the thermal properties are described by the mean-field theory if the order parameter is not conserved. We investigate the condition in detail, and confirmed this property. Moreover, we found that even in this case, the properties in a fixed value of order parameter cannot be described by the mean-filed theory in some parameter region. This indicates that the uniform configuration of the mean-field theory becomes unstable in such parameter region. We are studying the properties of such states. [11, 31, 51]

Phase transitions of the mixed phases

We have pointed out that the partially-disordered phase of the antiferromagnetic Ising model in the triangular lattice is a kind of mixed phase of a generalized six-states clock model.The mixed state is an equilibrium phase in which two of the six states are chosen to appear. We have studied general structure of the mixed states as a function of energy structure of the interaction. We have demonstrated a mixed phase with more than two states, and also successive phase transitions with different types of mixing. In Fig. 1.1.3, we show a temperature dependence of populations of the states. There we find a disordered phase at high temperature where all the six states have the same population, and then a phase of a 3-phase-mixing phase and then 2-state-mixing phase and finally a ferromagnetic phase (single state) as the temperature decreases.[20]

Stochastic process

Generalization of many particle Brownian motion has been proposed by using a differential-difference operator so-called Dunkl operator. Processes given by the operator is called Dunkl processes and have been studied in the field of mathematics. We have studied explicit expression of the effect of the intertwining operator. The processes are deeply related to the Dyson's Brownian motion, and we have studied relations of them to physical processes.[36, 42, 52]

1.2 Quantum Statistical Mechanics

Cooperative phenomena in quantum systems are also important subject in our group. In quantum systems, they show interesting non-classical behavior both in static and dynamical properties. In the last year, we studied the following topics.

Qauntum phases

We studied ground and low temperature properties of antiferromagnetic Heisenberg model on the Kagome lattice. We investigated effects of types of Dzyaloshinskii-Moriya Interactions and also effects of distributions of the spin length(i.e., $S = 1/2$ and 1).[5]

We also propose an itinerant electrons model (Hubbard model) in which the total spin is controlled by the chemical potential, and proposed new types of molecular magnets[2]

Qauntum respose

We also studied the dynamical properties and also response where various interesting processes appear.[2] Coherent dynamics of quantum systems exhibits various nonclassical natures and the manipulation of such processes gives important basis of quantum information processing. We have developed formulations of the quantum master equation to describe quantum response in dissipative environments.

In the last year, we studied hybridization of a system with discrete energy structure (spins or atoms) in the cavity and the cavity photon. We studied how the nature of the system changes with the number of spins and also as a function of the strength of driving force (intensity of input field). We clarified how the system move from the weakly excited region where we observe the vacuum-field Rabi splitting to the strongly excited region where we observe the Rabi oscillation in the classical electromagnetic field.[10] We show the dependence of Rabi oscillation on the number of photons in the cavity in Fig. 1.1.4.

We provide numerical tool for super-computer to calculate the dynamics of quantum master equation (Portal site for Application Software Library: quantum-dynamics-simulator)[56].

We also developed a new master equation to study the cases with strong interaction between spins and photons, where interesting cooperativity appear. When the interaction becomes strong, the ground state of the system exhibits a phase transition and photon and polarization appear spontaneously which is called Dicke transition. Beside this transition, it is known that the system exhibits a nonequilibrium phase transition under driving force, which is called optical bistability. There, due to a change of balance between driving force and dissipation, a discontinuous changes of quantities in the stationary state take place. We studied the synergetic effects of the both phase transition, and obtained phase diagram as a function of the interaction between spin and photon and the strength of the driving force. In order to study dissipative phenomenon in strongly interacting system, we need to extend the master equation from the simple Lindblad form to ones in which effects of interaction are taken into account in dissipative mechanism. We built up such equation of motion and obtained the phase diagram for the Tavis-Cummings model and alos for Dicke mode. [37, 43]

Dissipation of the Rabi oscillation

The Rabi oscillation has been measured as a prove of quantum coherence of spins (or any discrete energy level system). We have studied mechanism of decoherence due to the randomness of the parameters for each spin such as distribution of magnetic anisotropy and strength of the magnetic field, and also due to the dipolar-dipolar interaction by using large scale computation. [7]

We also propose an experiment to check the picture of wavefunction collapse in individual events in quantum mechanics. [8]

Quantum inverse scattering method for higher spin systems

As a study on the exact solvable models, we studied the exact property of spin chain by making use of algebraic Bethe anzatz. In particular, we investigated properties of boundary states of $S = 1$ spin chain, and studied effects of boundary condition on the ground state in quantum integrable systems. We also clarified the relation between nonlinear equation and the supersymmetric sine-Gordon model. [19]

Quantum transport phenomena

Classification of fluctuations in nonequilibrium statistical mechanics has been developed extensively. The so-called fluctuation theorem is one of the typical example. We have studied to verify the fluctuation theorem in quantum transport phenomena.[12] Moreover, the so-called additivity principle is also important property and we have extend the idea.[15] We also studied exact properties of the stationary nonequilibrium states in heat conducting quantum systems.[14, 16] We also studied general properties of the thermodynamical efficiency in micro systems. [13, 17]