Ground-State Statistics from Annealing Algorithms: 
Quantum vs Classical Approaches

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We study the performance of quantum annealing for systems with ground-state degeneracy by directly solving the Schrödinger equation for small systems and quantum Monte Carlo simulations for larger systems. While simulated annealing (SA) uses the classical thermal fluctuations to obtain the ground state of a target Hamiltonian (cost function), quantum annealing (QA) uses quantum fluctuations. An extensive body of numerical as well as analytical studies show that QA is generally more efficient than SA for the ground-state search of classical Hamiltonians of the Ising type. However, almost all problems studied with QA have been for nondegenerate cases, and researchers have not paid particular attention to the role played by degeneracy. This question needs careful scrutiny because many practical problems have degenerate ground states. If we are asked to identify all (or many of) the degenerate ground-state configurations and not just the lowest value of the energy, we have to carefully check if all ground states can be found. This would thus mean that the chosen algorithm can reach all possible ground-state configurations ergodically.

The results of our study indicate that naive quantum annealing using a transverse field may not be well suited to identify all degenerate ground-state configurations, although the value of the ground-state energy is often efficiently estimated. An introduction of quantum transitions to all states with equal weights is shown to greatly improve the situation but with a sacrifice in the annealing time. We also clarify the relation between the spin configurations in the degenerate ground states and the probabilities that those states are obtained by quantum annealing. The strengths and weaknesses of quantum annealing for problems with degenerate ground states are discussed in comparison with classical simulated annealing.